NOTE.

Some Formulas Concerning Surface Tension.—In their tabulation of measurements of surface tension Harkins and Brown[1] have used two alternative formulas connecting the weight of a drop with surface tension and other properties of the liquids. One of these is understood as chiefly empirical in origin; the other was previously deduced by Lohnstein on the basis of a solution by mechanical quadrature of the differential equation of the meridian curve, but since his theory included an assumption that seemed possibly questionable it was judged best to determine experimentally the form of the function entering. The object of this note is to point out that without a detailed dynamical theory the principle of dynamical similitude alone leads to equations of those two types and also suggests certain extensions of them which may be needed in more general cases.

Application of the method of similitude requires a list of the magnitudes entering into the relations sought. It is naturally not possible to make this list complete when indefinite precision is sought, but the value of the method appears in the fact that in general a certain few quantities only are of major importance in determining the variations. In such a case as the present one, where the underlying theory is taken to be purely dynamical, the condition of similitude implies that a relation between the n salient magnitudes must be capable of expression in the form of a relation between not more than $n - 3$ independent dimensional invariants, as they may be called, or products of powers of those magnitudes such as to be dimensionless under arbitrary changes in length-, time-, and mass-factors.

If it be supposed first that the required relation involves only the mass M of the drop, the surface tension γ, the gravitational field strength g, the density D of the drop, and the radius r of the measuring tip, making five quantities, then two invariants are required. These may be taken as

$$Mg/2\pi r\gamma = z, \; r/a = x,$$

where a is a length defined by $a^2 = 2\gamma/gD$. This supposition implies that the shape of the drop or at least its volume is determined by the same quantities, and since $r/b = y$ say is also dimensionless, where $b^3 = V$, there must also be a relation between x and y, so that the formula initially sought can be thought of in the two alternative forms

$$z = f(x) = \psi(y) \qquad (1)$$

which are equivalent to those used by the authors named. To accompany these the relation between x and y may be written in the two forms

$$y^3 = x^2/\pi f(x), \; x^2 = \pi y^3 \psi(y) \qquad (2)$$

[1] THIS JOURNAL, 41, 499 (1919).

which result if Db^3 be substituted for M in the definition of z and the result written in terms of a and b. These show how the (z,x) and (z,y) curves are such, that though so far as the principle of similitude is concerned the form of one of these functions is indeterminate, that of the other is determinately related to it. As a third alternative the relation between x and y could be treated as the single undetermined one, to be found experimentally or by a more detailed theory. This relation is also tabulated in the results by Harkins and Brown.

More symmetrically, one may view (2) as merely a transcription of the identical relation

$$x^2 = \pi z y^3 \tag{3}$$

between the three invariants. Geometrically this may be thought of as represented by the corresponding surface with (x, y, z) as coördinates, then the physical relation sought is represented by a certain curve on that surface, subject to determination for example by any one of its three projections. Consistency between these various determinations of the same space curve is a test of justification of the assumptions hitherto made, a consistency which appears to be closely reached in the measurements, though there are some suggestions of systematic departures. Since these assumptions practically amount to supposing the phenomena essentially statical, it is natural to take for D the difference of densities of the drop and the surrounding fluid.

The rupture leading to the escape of the drop is, however, to some extent a kinetic process, as appears from its rather startling suddenness. Thus it is possible that the ratio, say u, of the densities might also enter, or in other words the two densities might need to be regarded as entering as distinct variables. In that case the relation (1) would be replaced by one of the form

$$z = f(x,u) = \psi(y,u) \tag{4}$$

and the relation between x and y would probably also involve u. If then the data for cases including various values of the ratio of densities were combined in plotting according to the scheme (2) there might be departures from the single curve previously considered, though from the observations now in hand this effect appears to be small. It seems possible that viscosity also might have some influence, through its effect on conditions of stability.

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